Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Friday 5 May 2017 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

• Write your session number in the boxes above.
• Do not open this examination paper until instructed to do so.
• A graphic display calculator is required for this paper.
• Section A: answer all questions. Answers must be written within the answer boxes provided.
• Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
• Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
• A clean copy of the **mathematics SL formula booklet** is required for this paper.
• The maximum mark for this examination paper is **[90 marks]**.

12 pages

12EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1.   [Maximum mark: 7]

   Consider the following frequency table.

x	Frequency
2	8
4	15
7	21
10	28
11	3

(a)   (i)   Write down the mode.

   (ii)   Find the value of the range.   [3]

(b)   (i)   Find the mean.

   (ii)   Find the variance.   [4]

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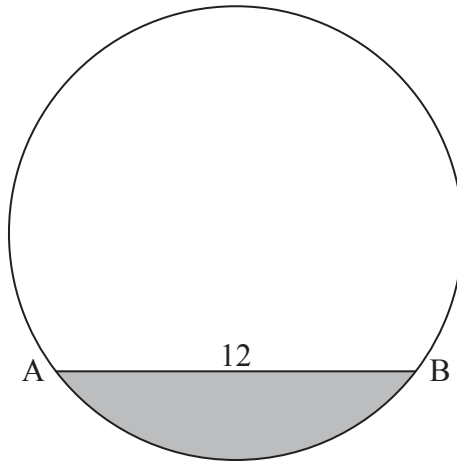
12EP02

2. [Maximum mark: 6]

Let $v = \begin{pmatrix} -10 \\ 2 \\ 1 \end{pmatrix}$ and $w = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$. Find the angle between v and w, giving your answer correct to one decimal place.

Turn over

3.   [Maximum mark: 6]

Consider the graph of $f(x) = \dfrac{e^x}{5x - 10} + 3$, for $x \neq 2$.

(a)   Find the y-intercept.   [2]

(b)   Find the equation of the vertical asymptote.   [2]

(c)   Find the minimum value of $f(x)$ for $x > 2$.   [2]

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4. [Maximum mark: 6]

In a large university the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let k be the expected number of left-handed students in this sample.

(a) Find k. [2]

(b) Hence, find the probability that

(i) exactly k students are left handed;

(ii) fewer than k students are left handed. [4]

Turn over

5.   [Maximum mark: 7]

The following diagram shows the chord [AB] in a circle of radius $8\,\text{cm}$, where $\text{AB} = 12\,\text{cm}$.

diagram not to scale

A   12   B

Find the area of the shaded segment.

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6.   [Maximum mark: 7]

Let $f(x) = (x^2 + 3)^7$. Find the term in x^5 in the expansion of the derivative, $f'(x)$.

Turn over

7. [Maximum mark: 7]

A particle P moves along a straight line. Its velocity $v_P \, \mathrm{m\,s^{-1}}$ after t seconds is given by $v_P = \sqrt{t} \sin\left(\dfrac{\pi}{2} t\right)$, for $0 \leq t \leq 8$. The following diagram shows the graph of v_P.
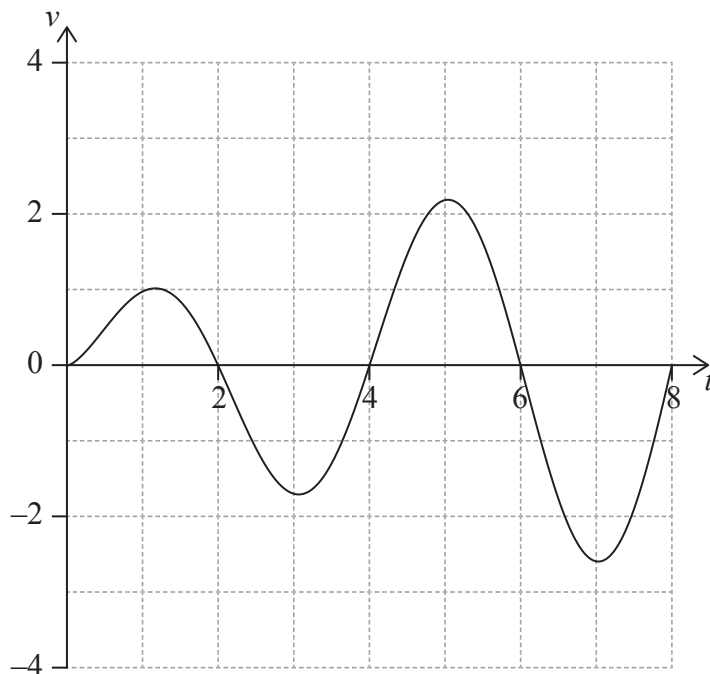
(a) (i) Write down the first value of t at which P changes direction.

(ii) Find the **total** distance travelled by P, for $0 \leq t \leq 8$. [3]

(b) A second particle Q also moves along a straight line. Its velocity, $v_Q \, \mathrm{m\,s^{-1}}$ after t seconds is given by $v_Q = \sqrt{t}$ for $0 \leq t \leq 8$. After k seconds Q has travelled the same total distance as P.

Find k. [4]

(This question continues on the following page)

(Question 7 continued)

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

At Grande Anse Beach the height of the water in metres is modelled by the function $h(t) = p\cos(q \times t) + r$, where t is the number of hours after $21{:}00$ hours on 10 December 2017. The following diagram shows the graph of h , for $0 \le t \le 72$.
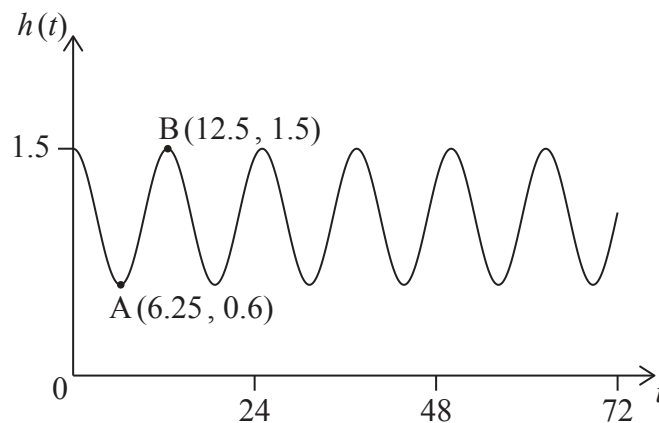

The point $A(6.25, 0.6)$ represents the first low tide and $B(12.5, 1.5)$ represents the next high tide.

(a) (i) How much time is there between the first low tide and the next high tide?

  (ii) Find the difference in height between low tide and high tide. [4]

(b) Find the value of

  (i) p ;

  (ii) q ;

  (iii) r . [7]

(c) There are two high tides on 12 December 2017. At what time does the second high tide occur? [3]

9.   [Maximum mark: 15]

A random variable X is normally distributed with mean, μ. In the following diagram, the shaded region between 9 and μ represents 30% of the distribution.
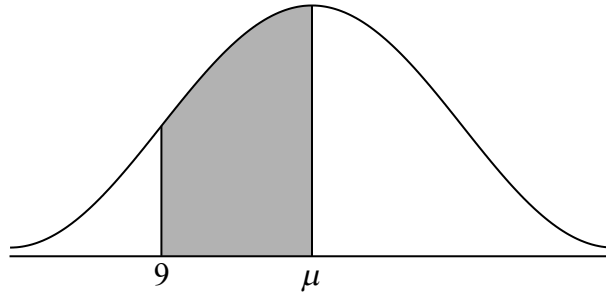
(a)   Find $P(X < 9)$. [2]

The standard deviation of X is 2.1.

(b)   Find the value of μ. [3]

The random variable Y is normally distributed with mean λ and standard deviation 3.5. The events $X > 9$ and $Y > 9$ are independent, and $P\big((X > 9) \cap (Y > 9)\big) = 0.4$.

(c)   Find λ. [5]

(d)   Given that $Y > 9$, find $P(Y < 13)$. [5]

Turn over

10. [Maximum mark: 15]

Let $f(x) = \ln x$ and $g(x) = 3 + \ln\left(\dfrac{x}{2}\right)$, for $x > 0$.

The graph of g can be obtained from the graph of f by two transformations:

<div style="text-align:center">

a horizontal stretch of scale factor q followed by

a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$.
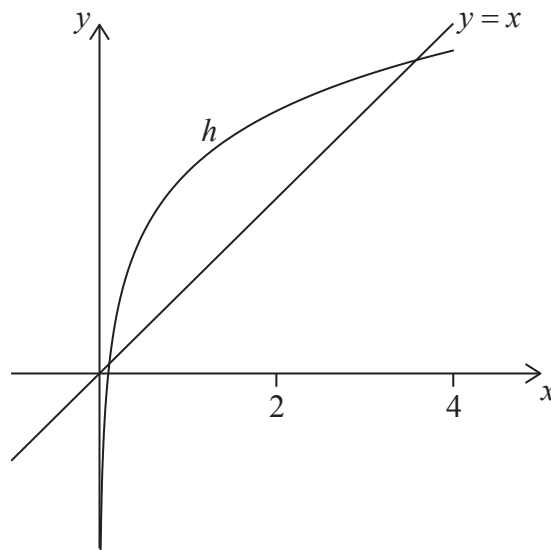
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(a)   Write down the value of

   (i) q;

   (ii) h;

   (iii) k.   [3]

Let $h(x) = g(x) \times \cos(0.1x)$, for $0 < x < 4$. The following diagram shows the graph of h and the line $y = x$.

The graph of h intersects the graph of h^{-1} at two points. These points have x coordinates 0.111 and 3.31, correct to three significant figures.

(b)   (i)   Find $\displaystyle\int_{0.111}^{3.31}\left(h(x) - x\right)\mathrm{d}x$.

   (ii)   Hence, find the area of the region enclosed by the graphs of h and h^{-1}.   [5]

(c)   Let d be the vertical distance from a point on the graph of h to the line $y = x$. There is a point $\mathrm{P}(a, b)$ on the graph of h where d is a maximum.
Find the coordinates of P, where $0.111 < a < 3.31$.   [7]